Calculation Policy

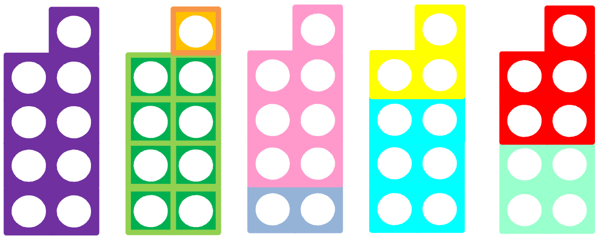
Stages in counting

All children go through these stages in counting. Generally, they should be secure with them by the end of Year R.

1. Stable order (knowing numbers come in an order)
2. One to one correspondence (touching and counting)
3. Cardinal (knowing last number is the total)
4. Order irrelevance (doesn't matter how you count the total will be the same)
5. Abstraction (being able to count without seeing/touching items)

By the end of Year R children should also be able to:

- Subitise (know number of dots on dice or dominoes without counting)
- Know about the numbers to 10, for example, 9 is made up of 1 and 8, 2 and 7 etc., it is greater than 4, less than 10 (Numicon is great for this)



- Recognise and begin to write numerals

Key mental calculation strategies

It is important to spend time developing mental calculation strategies so that children have a bank of them to use when calculating. Often a calculation can be answered more efficiently using these and yet most children in KS2 will use a written method. Spend a week or maybe two developing these before working on written methods.

Some of these are started in EYFS and Year 1 and developed in subsequent years. They include:

From Year R/1:

- Partition and recombine one number to 20,
  e.g. 14 = 10 + 4
- Doubles and near doubles,
  e.g. 6 + 6 = 12, 25 + 26 = double 25 add 1
- Use number pairs to 10 and 100,
  e.g. 3 + 7 = 10, 40 + 60 = 100, 25 + 75 = 100, 63 + 37 = 100
- Counting on to subtract numbers close together,
  e.g. 26 – 17: + 3 + 6 = 9, 2136 – 1989: + 11 + 136 = 147

Keep practising and developing the above. Additional strategies from Year 1/2:

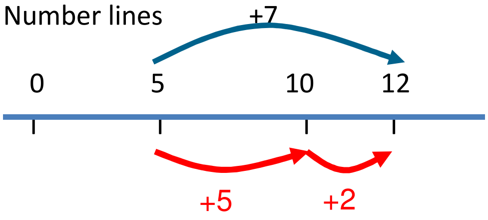
- Adding near multiples of ten and adjusting,
  e.g. 145 + 199 = 145 + 200 – 1 = 344, 1587 + 1997 = 1587 + 2000 – 3 = 3584
- Sequencing,
  e.g. 256 + 153 = 256 + 100 + 50 + 3 = 409
- Using patterns of similar calculations,
  e.g. 136 + 45 = 181, 146 + 45 = 191, 156 + 45 = 201
- Using known number facts,
  e.g. 6 x 7 = 42, 6 x 70 = 420, 6 x 35 = 210
- Bridging though ten, hundred, tenth,
  e.g. 26 + 15 = 30 + 11 = 41, 125 + 76 = 130 + 71 = 201
- Use relationships between operations,
  e.g. 4 x 5 = 20, 5 x 4 = 20, 20 ÷ 5 = 4, 20 ÷ 4 = 5
- Making a subtraction easier (same difference, different calculation),
  e.g. 27 – 18 becomes 29 - 20

Keep practising and developing the above. Additional strategies from Year 3:

- Regrouping for division,
  e.g. 132 ÷ 3: 120 ÷ 3 = 40 and 12 ÷ 3 =4
- x4 by doubling and doubling again,
  e.g. 36 x 4 = 72 x 2 = 144
- x5 by x10 and halving or vice versa,
  e.g. 346 x 5 = 3460 ÷ 2 or 173 x 10 = 1730
- x20 by x10 and doubling,
  e.g. 427 x 20 = 4270 x 2 = 8540
- x15 by x 10, halve and ad,
  e.g. 135 x 15: 1350 + 675 = 2025
- ÷4 by halving and halving,
  e.g. 120 ÷ 4 = 60 ÷ 2 = 30
- ÷5 by dividing by 10 and doubling,
  e.g. 375 ÷ 5 = 37.5 x 2 = 75
- ÷20 by dividing by 10 and halving,
  e.g. 246 ÷ 20 = 24.6 ÷ 2 = 12.3

Models and images for mental calculation strategies that children *may* find helpful.

Number lines

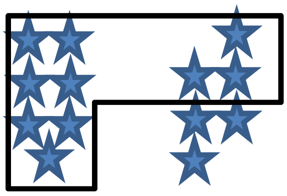


Partitioning for sequencing

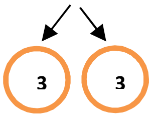
48 + 33

30 3 = 78 + 3 = 81 48 + 30 + 3 = 78 +3 = 81

Bridging 10

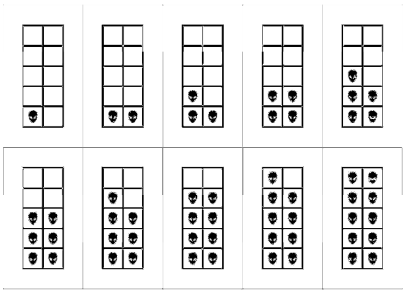


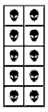
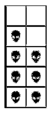
7 + 6= 13

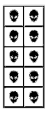


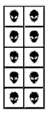
10 + 3 = 13

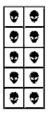
Tens frames for number pairs to 1, 10, 100, multiplication

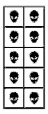


If  represents 10, show me the frame that goes with  to make 10. What calculations can we make? 7 + 3 = 10 and 3 + 7 = 10 because addition is commutative.

What if  represents 100?

What if  represents 1? 0.1? This is a good activity for number facts.

If  represents 40, show me 32. What calculations can we make? 8 x 4 = 32, 4 x 8 = 32 because multiplication is commutative.

If  represents 90, show me 108 and so. This is a good way to rehearse multiplication facts.

Multiplication grids to highlight the commutative property of multiplication - we only need to learn half the facts, because in learning one we know two, e.g. if we know 9 x 4 = 36, we also know 4 x 9 = 36.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2		4	6	8	10	12	14	16	18	20	22	24
3			9	12	15	18	21	24	27	30	33	36
4				16	20	24	28	32	36	40	44	48
5					25	30	35	40	45	50	55	60
6						36	42	48	54	60	66	72
7							49	56	63	70	77	84
8								64	72	80	88	96
9									81	90	99	108
10										100	110	120
11											121	132
12												144

Focus on square numbers, learn these facts and link in with areas of squares – practically. Teach 3s and 6s together, 4s and 8s together, 6s and 12s together because one is double the other.

Place value

- Positional: the quantities represented by the individual digits are determined by the positions that they hold in the whole numeral. The value given to a digit is according to the position in a number
- Base 10: the value of the position increases in powers of 10
- Multiplicative: the value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position
- Additive: the quantity represented by the whole numeral is the sum of the values represented by the individual digits

For example:

1000	100	10	1
6	8	3	7

Positional: 6 is in the 1000's position, 8 in the hundreds, 3 in the tens and 7 in the ones
Multipicative: because 6 is in the 1000's position it is multipled by 1000 to give its value, 8 is multiplied by 100, 3 by 10 and 7 by one to give their values.
Additive: add all the numbers together to give the total 6000 + 800 + 30 + 7 = 6837
Base 10: if 37 is multipled by 10, 3 tens becomes 3 hundreds, 7 ones become 7 tens and a place holder is placed in the ones position: 370

2 9r2
6|1256

The children need to be able to partition numbers in different ways to help understanding and also to encourage mental calculation strategies:

- Partition all pairs of numbers for all numbers to 20, e.g. 1 + 4 = 5, 2 + 3 = 5, 3 + 2 = 5

- Partition 2-, 3- etc. digit numbers in different ways, e.g. 57: 50 + 7, 40 + 17, 30 + 27, 20 + 17, 10 + 7

Equals sign

The equals sign is not an indication of an answer. It is a sign of equivalence – the same as.

Year 1: 2 + 3 = 1 + ⌂

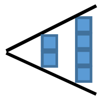
Year 5: 23 + y = 35. Take 23 away from both sides, y = 12

Year 6: 2y + 36 = 40. Take 36 away from both sides, 2y = 4. Divide both sides by 2, y = 2

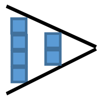
Greater than and less than

Be mathematical when teaching this (no crocodiles)

Show these symbols plus equals like this (good to have on the wall):

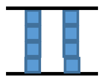


2 < 4

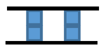


4 > 2

How can we make these equal?



2 + 2 = 4



2 = 4 - 2

Addition and subtraction

National Curriculum requirements

Year 1
Add and subtract one-digit and two-digit numbers to 20, including zero using concrete objects, pictorial representations and mentally
Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ▯ - 9

Year 2
Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
* a two-digit number and ones
* a two-digit number and tens
* two, two-digit numbers
* adding three one-digit numbers
Solve problems with addition and subtraction:
* using concrete objects and pictorial representations, including those involving numbers, quantities and measures
* applying their increasing knowledge of mental and written methods

Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change (copied from Measurement)

Year 3
Add and subtract numbers mentally, including:
* a three-digit number and ones
* a three-digit number and tens
* a three-digit number and hundreds

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Year 4
Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

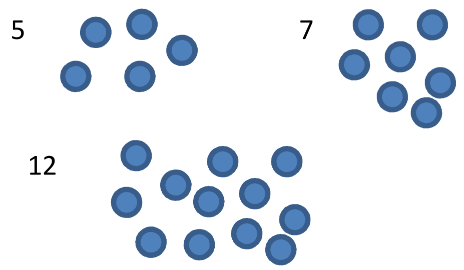
Year 5
Add and subtract numbers mentally with increasingly large numbers
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Year 6
Perform mental calculations, including with mixed operations and large numbers
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
Solve problems involving addition, subtraction, multiplication and division

Begin by teaching children the relationship between addition and subtraction – commutativity and inverse. When teaching addition use subtraction as a check. When teaching subtraction use addition as a check. Use calculators for checking!

Structures for addition

Aggregation (counting all)



Augmentation (adding on to a set)

Vocabulary: augend add addend equals sum, commutative

Progression towards the written method

Year R: Use counters, bead strings, any 'stuff' and progress to Numicon to encourage counting on from one number to find the sum of quantities to 10 and, if appropriate, to 20.

Year 1: Continue using Numicon, finding the two numbers putting them together and finding the sum without counting everything to 20 and, if appropriate, to 50.

Combine tens and ones using Dienes or bundles of straws and write number statements/draw pictures to show what they have done. Towards the end of Year 1 explore exchange.

Year 2: Continue combining Dienes to make sums up to 100, including exchange. Writing number statements/drawing pictures to show what they have done.

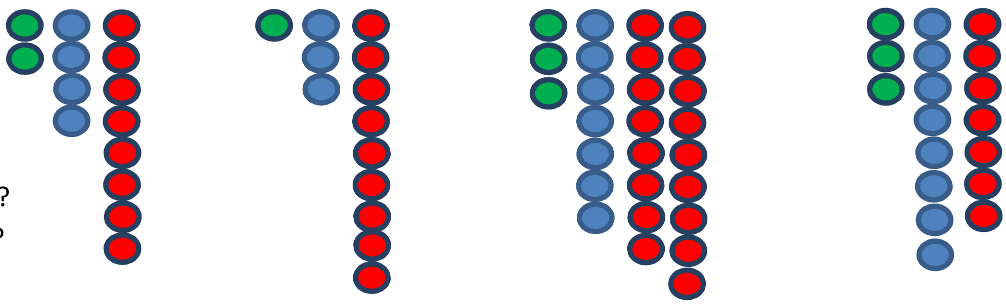
Towards end of the year introduce vertical partitioning to prepare children for Year 3.

48
+ 33
70
11
81

Year 3 and above: Use of manipulatives to lead to the written method. Important to explore why they need to begin with the least significant number.

Be careful to use 3-digit examples that cannot be answered using a mental method.

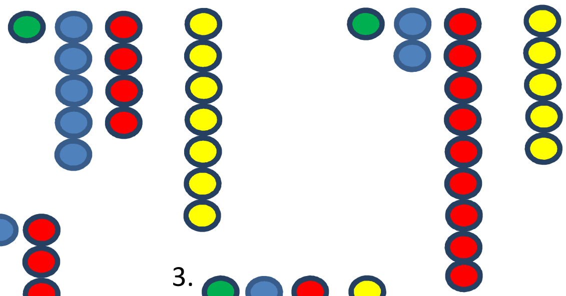
2 4 8	leading to	2 4 8	with manipulatives
+ 1 3 9		+ 1 3 9	
3 0 0		3 8 7	
7 0		1	
1 7			
3 8 7			



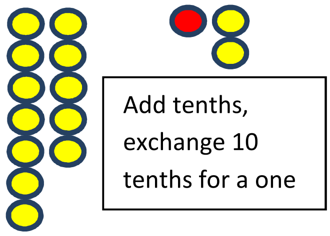
What's the same?
What's different?

Written method: decimals (with manipulatives first)

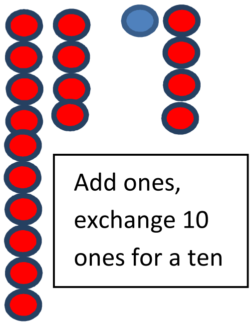
```
  154.7
+ 129.5
  284.2
   1 1
```



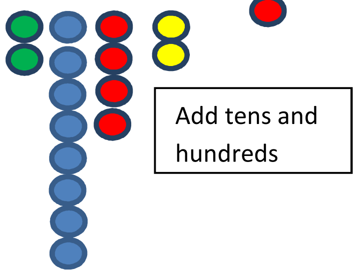
1.



2.



3.

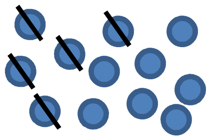


Every time new numbers are introduced, e.g. thousands, decimal places, use manipulatives first so children can see that the process is essentially the same.

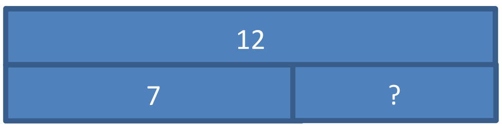
Structures for subtraction

Subtraction (take-away)

12 – 5 = 7



Difference (comparison model)



Reduction (more abstract: temperature, speed)

Progression towards the written method

Vocabulary: minuend subtract subtrahend equals difference

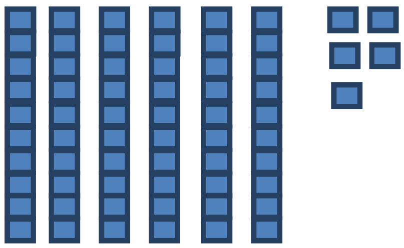
Year R: Use counters, bead strings, any ‘stuff’ to find the difference between quantities to 10 and, if appropriate, to 20.

Year 1: Continue using the manipulatives from Year R and also Numicon, to find the difference between quantities to 20 and, if appropriate, to 50.

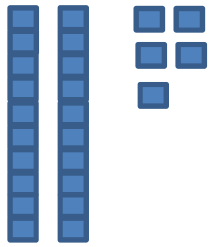
Make a number using Dienes or bundles of straws and subtract a smaller number. Write number statements/draw pictures to show what they have done. Towards the end of Year 1 explore exchange.

Year 2: Continue using Dienes to find differences between quantities to 100, including exchange. Writing number statements/drawing pictures to show what they have done.

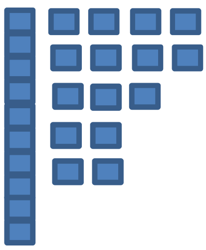
65 – 47

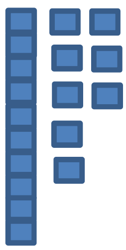


Take away 40



Exchange one 10 for 10 ones in order to take away 7



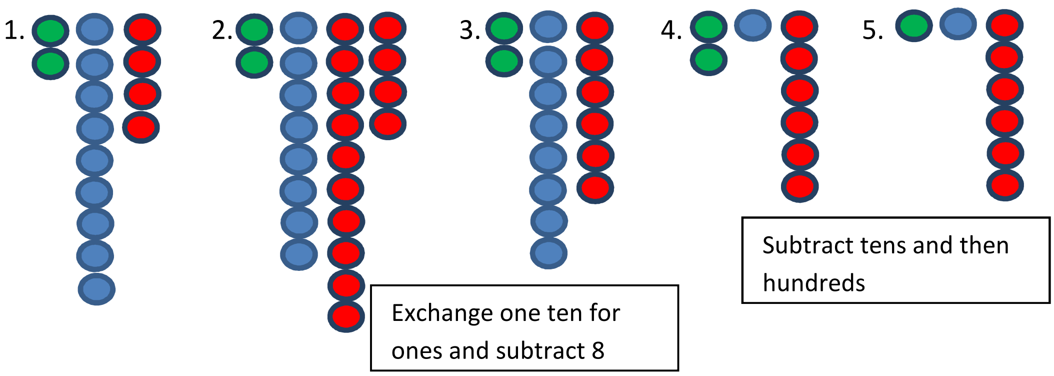


Year 3 and above: Use of manipulatives to lead to the written method by the end of Year 3, beginning of Year 4.

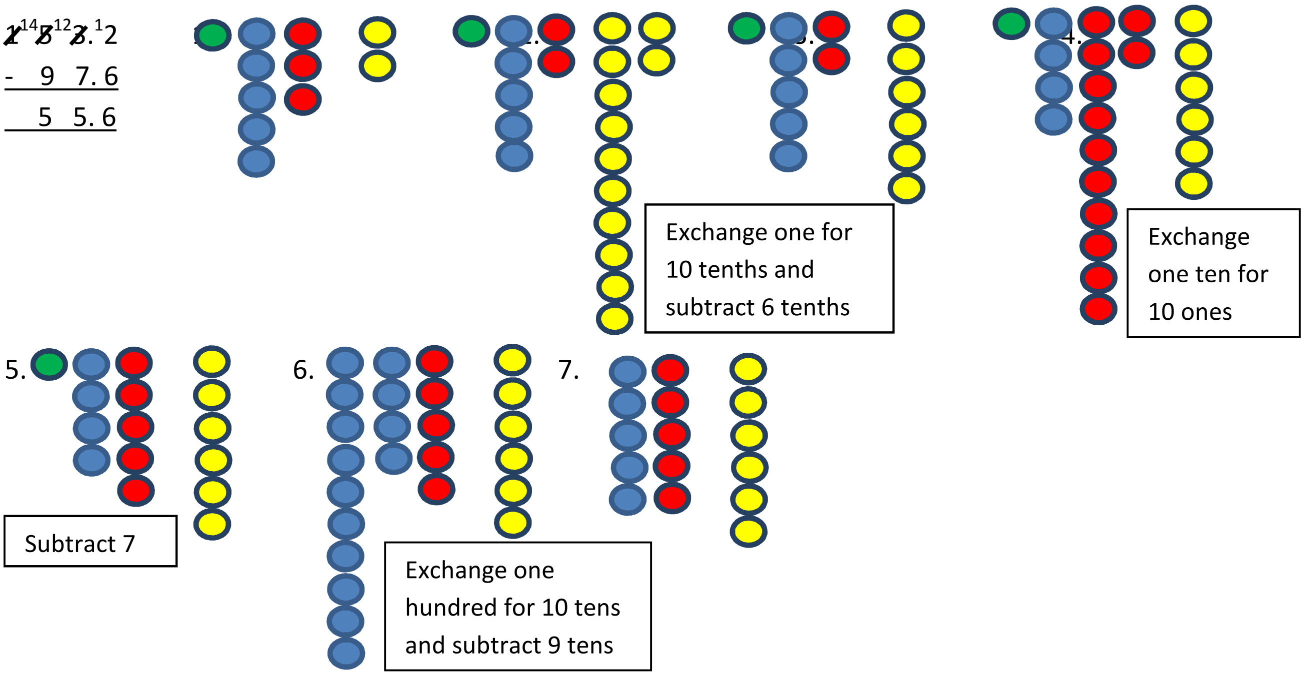
<u>Be careful to use 3-digit examples that cannot be answered using a mental method.</u>

Use of manipulatives to lead to compact method:

  2 ^{8}~~9~~^{1}4
- 1 7 8
  1 1 6



Written method: decimals (with manipulatives first)



Multiplication and division

National Curriculum requirements

Year 1

Count in multiples of twos, fives and tens (copied from Number and Place Value)

Year 2

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

Year 3

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Year 4

Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Year 5

Multiply and divide numbers mentally drawing upon known facts

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Year 6
Perform mental calculations, including with mixed operations and large numbers
Solve problems involving addition, subtraction, multiplication and division
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
Use written division methods in cases where the answer has up to two decimal places (copied from Fractions (including decimals)

Begin by teaching children the relationship between multiplication and division – commutativity and inverse. When teaching multiplication use division as a check. When teaching division use multiplication as a check. Use calculators for checking!

Link multiplication to repeated addition and division to repeated subtraction. Both are grouping. Link the sharing model for division to fractions.

Structures of multiplication

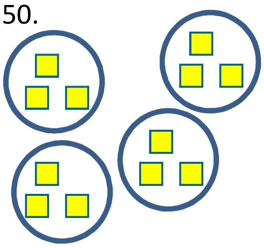
Grouping (repeated addition)
Scaling (2, 3, 4 etc. times as many)

Progression towards the written method

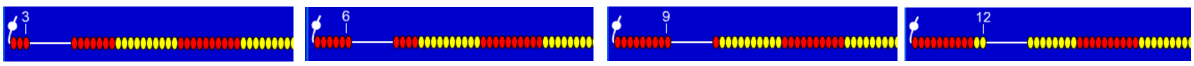
Vocabulary: multiplicand multiplied by multiplier equals product, commutative

Year R: Use counters, bead strings, any 'stuff' to groups of quantities to 10 and, if appropriate, to 20.

Year 1: Continue using the manipulatives from Year R and also Numicon, to find groups of quantities, e.g. 2, 5 and 10 to 20 and, if appropriate 50.

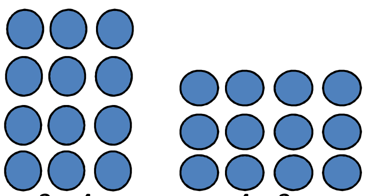


4 groups of 3 = 3 four times = 3 x 4
Bead strings



Set counters out as arrays and explore commutativity and early inverse by taking groups away.

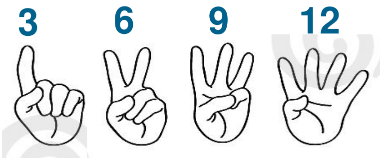
Arrays



3 x 4   4 x 3

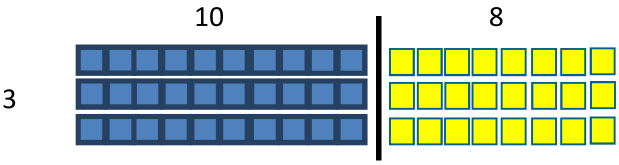
Year 2: Continue as in Year 1, focussing on arrays, include multiplying by 3 and writing the commutative number statements.

Fingers



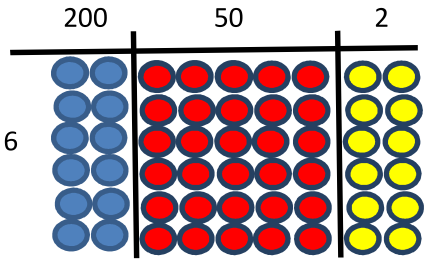
Year 3 and above: Use arrays and link to the grid methods, beginning with 2-digit multiplicands.
<u>Be sure to use multipliers that do not lend themselves to a mental method, e.g. 2, 4, 5, 10</u>.
These multipliers are suitable: 3, 6, 7, 8 and 9.

Arrays to support the grid method



	10	8
3	30	24

18 x 3 = 54

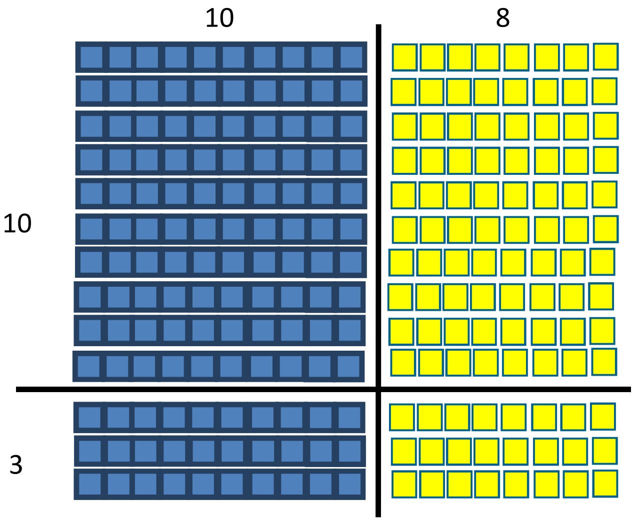


	200	50	2
6	1200	300	12

= 1512

Or…	leading to written method
252	252
x 6	x 6
12	1512
300	3 1
1200	
1512	

Long multiplication begins in Year 5:



Also use Dienes and place value counters to demonstrate this

	10	8	
10	100	80	180
3	30	24	54
		Total:	234

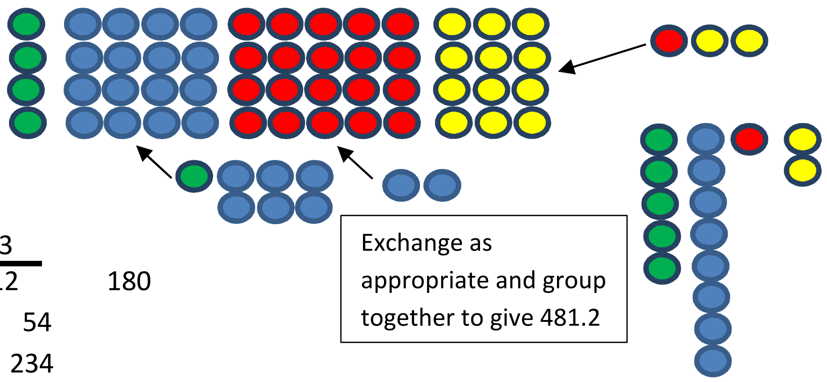
Leading to written method:

```
  18
x 13
  54
 180
 234
```

or…

```
  18      18      180
x 10     x 3    +  54
 180      54      203
```

Written method: decimals



	100	40	5	.3	
4	400	160	20	12	180
3	30		24	54	
			Total:	234	

```
145.3
x  4
581.2
 111
```

Structures for division

Grouping (repeated subtraction)
Scaling (a third, quarter, fifth etc. of the size)
Sharing (best linked to fractions)

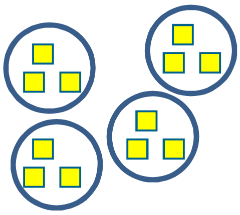
Progression towards the written method

Vocabulary: dividend, divisor, quotient, division bracket

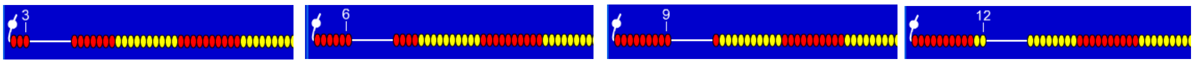
Year R: Use counters, bead strings, any 'stuff' to make quantities to 10, then20 into groups of 2s, 5s and 10s.

Year 1: Continue using the manipulatives from Year R and also Numicon, to find how many groups they can make out of quantities, to 20 and, if appropriate 50.

12 ÷ 3
How many groups of 3 in 12?

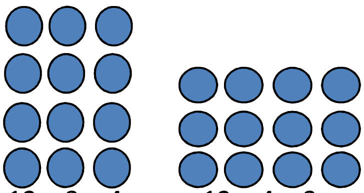


Bead strings



Set counters out as arrays and explore taking groups away.

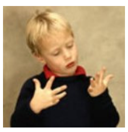
Arrays

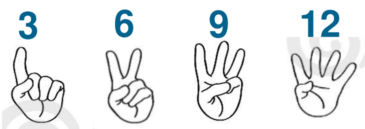


12 ÷ 3 = 4   12 ÷ 4 = 3

Year 2: Continue as in Year 1, focussing on arrays and working out division calculations by counting in step on fingers.

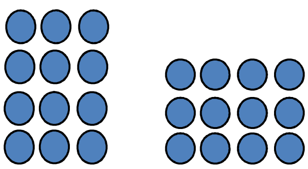
Fingers



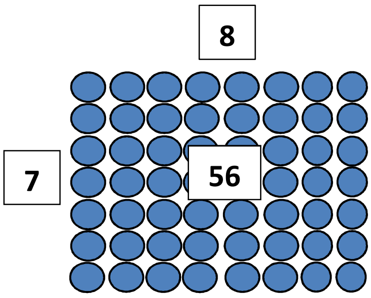


Year 3 and above: Use arrays and then move towards the written method using manipulatives.

Arrays

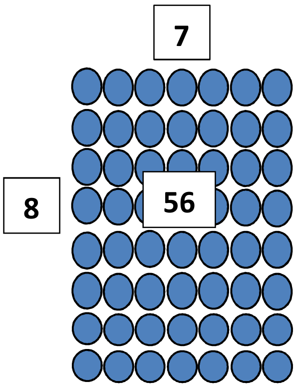


12 ÷ 3 = 4   12 ÷ 4 = 3

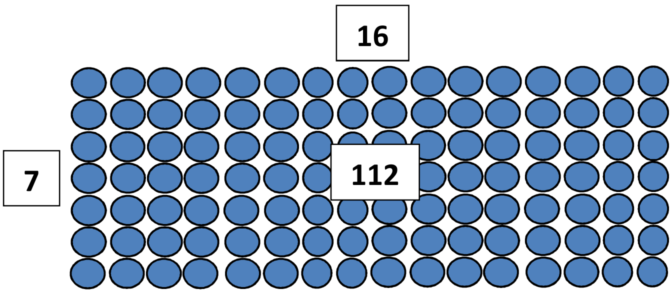


Introducing the conceptual variation of the written method with division bracket.

$$7\overline{)\,56\,}^{\;8}$$



$$8\overline{)\,56\,}^{\;7}$$



From this we know:

7 x 16 = 112

16 x 7 = 112

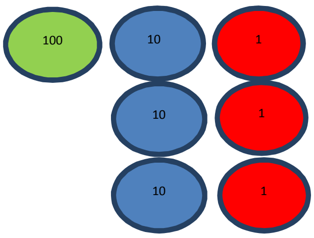
112 ÷ 16 = 7

112 ÷ 7 = 16

Written method

133 ÷ 6

Important to let the children use manipulatives such as place value counters to explore exchange:



You cannot take 6 groups of 100 away from the one 100

Exchange the 100 for 10 tens so you have 13 tens

$$6\overline{)\,\not{1}\,{}^{1}3\;3\,}$$

You can now take two groups of 6 tens

2

6 | ~~1~~ $^{1}3$ 3

One groups of ten will be left. This is exchanged for 10 ones. You now have 13 ones.

2

6 | ~~1~~ ^{1}~~3~~ $^{1}3$

You can take another two groups of 6 ones from the 13 leaving a remainder of 1

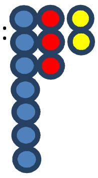
2 2 $^{1}/_{6}$

6 | ~~1~~ ^{1}~~3~~ $^{1}3$

Written method: decimals (with manipulatives first)

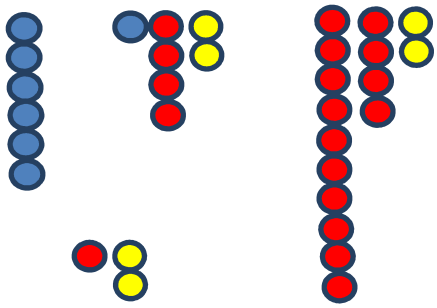
73.2 ÷ 6

Important to let the children use manipulatives such as place value counters to explore exchange:



You can take one group of six 10s away from the seven 10s. There will be one hundred left

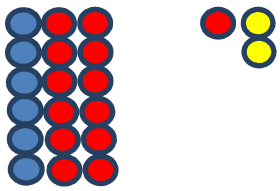
Exchange the 10 for 10 ones so you have 13 ones



1

6 | ~~7~~ $^{1}3$. 2

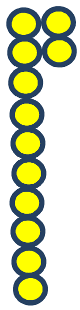
You can now take two groups of 6 tens



1 2

6 | ~~7~~ $^{1}3$. 2

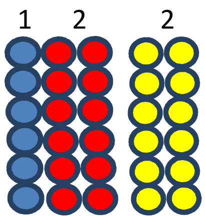
One will be left. This is exchanged for 10 tenths. You now have 12 tenths.



1 2. 2

6 | ~~7~~ ^{1}~~3~~. $^{1}2$

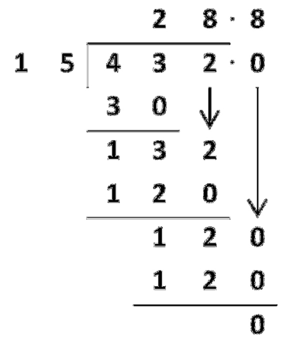
You can take two groups of 6 tenths



1 2. 2

6 | ~~7~~ ^{1}~~3~~. $^{1}2$

Long division appears in Year 6



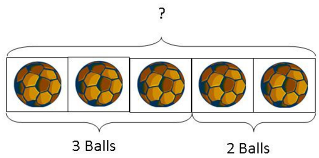
Answer: 28·8

The bar model

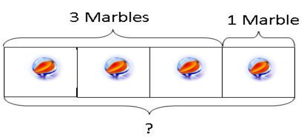
This visual representation helps children make sense of problems. It needs to begin in EYFS (practically and visually) then developed throughout the school. Use manipulatives for this whenever the children want to. Develop the drawings from these. Cuisenaire and double sided counters are good for this.

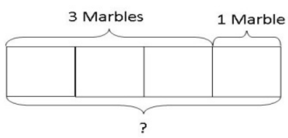
EYFS:

There are 3 footballs in the red basket and 2 footballs in the blue basket.
How many footballs are there altogether?



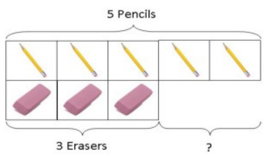
Peter has 3 marbles.
Harry gives Peter 1 more marble.
How many marbles does Peter have now?

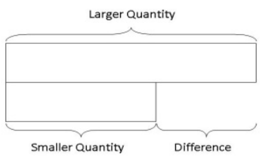




Visual to Abstract

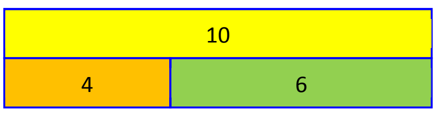
Peter has 5 pencils and 3 erasers.
How many more pencils than erasers does he have?



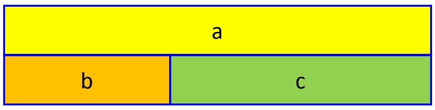


Visual to Abstract

Year 1 upwards:



This leads to an abstract model which helps with links between addition and subtraction



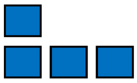
a = b + c	a = c + b
b = a – c	c = a - b

This can then help the children solve, for example, missing number problems:

45 + ? = 93, ? – 62 = 13, 146 - ? = 79, ? + 82 = 147

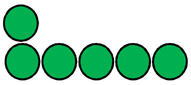
Peter has 4 books.
Harry has three times as many books as Peter. How many more books has Harry?

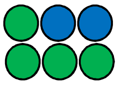
Peter's books
Harry's books



Sam had 5 times as many marbles as Tom. If Sam gives 26 marbles to Tom, the two friends will have exactly the same amount. How many marbles do they have altogether?

Tom's marbles
Sam's marbles





26 so each circle is worth 13

If the children had been proficient using this model, they would have found these 'Level 5' SATs questions simple:

24 In a class, 18 of the children are girls.

A quarter of the children in the class are boys.

Altogether, how many children are there in the class?

Show your working

A gardener plants tulip bulbs in a flower bed.

She plants 3 red bulbs for every 4 white bulbs.

She plants 60 red bulbs.

How many **white** bulbs does she plant?

They would have been able to attempt this level 6 question

4 Two numbers are in the ratio **4 : 5**

One of the numbers is **60**

There are two possible values for the other number.

What are the two possible values?

2 marks

And questions like these:
A shop keeper sold 1/3 of his balloons in the afternoon and 2/5 of the remainder in the evening.
If he had 150 balloons left, find the number of balloons he had at first?

Iqbal and Sofia have £680 altogether. If Iqbal spends
$\frac{2}{5}$ of her money and Sofia spends £80, then they will have an equal amount of money left. How much money did Sofia have at first?

Mr Yap had a length of rope. He used 1/4 of it to tie some boxes together. He then used 5/9 of the remainder to make a skipping rope for his daughter. 120cm of rope were left. What was the length of rope used to tie the boxes together?

Michelle prepared a mixture of apple, carrot and celery juices. 1/3 of the mixture was apple juice and 2/5 of the remainder was celery juice. 315 ml of the mixture was celery juice. What volume of the mixture was carrot juice?